[참고] Sectoral Price Facts in a Sticky-Price Model

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1 Empirical Sectoral Price Facts

□ The baseline FAVAR and structural model have **27 sectors**(partially combining 50 subcategories of the third-level diaggregation of PCE - (i) belong to a same second-level disaggregation & (ii) share a similar degree of nominal rigidities)

- Observables(quarterly): nominal interest rate(effective federal funds rate), aggregate consumption growth(real PCE), and inflation(PCE deflator).
- # common factors: 2, # lags = 4

1.1 Facts on the Speed of Price Responses to Different Shocks

□ The average speed of reponse of sectoral prices to sector-specific components(0.884) is substantially higher than to common shocks(0.283)

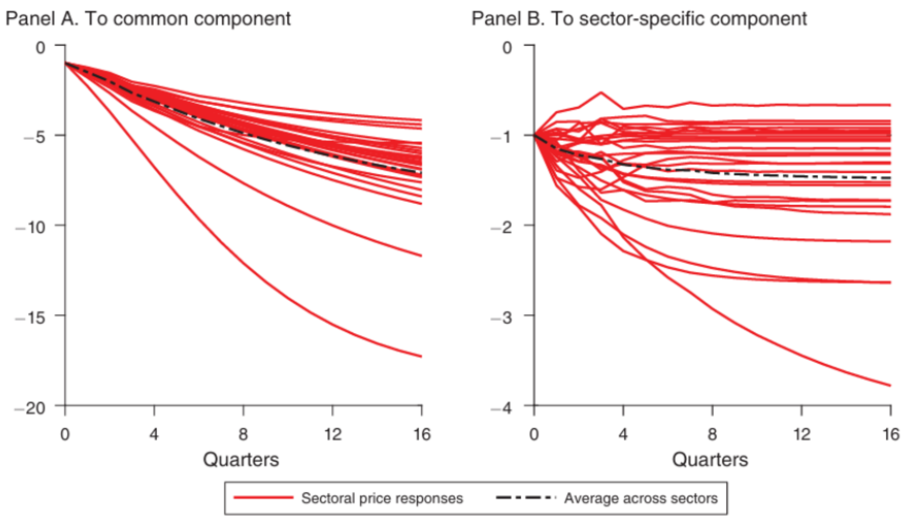


Figure 1. Impulse Response of PCE Price Series

- Relatively larger cross-sectional variation in the speed of resonses to sector-specific shocks(0.241) than to common shocks(0.046)
- Positive correlation between speeds of sectoral price responses and sectoral frequencies of price changes(0.460 for common components; 0.435 for sector-specific components)
- Positive correlation(0.363) between the speeds of responses to both types of shocks

1.2 Facts on the Correlations between Prices and Quantities

□ Negative correlation(-0.302/-0.143) between the sector-specific/common component of PCE inflation rates and the corresponding sector-specific/common component of quantities

- Supply-type shocks are relatively more important drivers of fluctuations in prices and quantities

2 The Model

- ☐ The model is a variant of the standard New Keynesian model with the following features:
  1. multiple sectors that differ in the degree of price stickiness and are subject to sectoral demand and supply shocks
  2. a roundabout input-output technology
  3. sector-specific labor markets
- ☐ In the model economy...
  - The economy is divided into a finite number of sectors; $k \in \{1, 2, \cdots, K\}$
  - There is a continuum of firms; $i \in [0, 1]$
  - Each firm belongs to one of the K sectors; $\mathcal{I}_k = \{i \in [0,1] : \text{firm } i \text{ belongs to sector } k\}$; $\bigsqcup_{k=1}^{K} \mathcal{I}_k = [0,1]$; $n_k = V(\mathcal{I}_k)$; $\sum_{k=1}^{N} n_k = 1$
  - Each firm produces differentiated good that is used for consumption and as an intermediate input

2.1 Representative Household

- ☐ The representative consumer...
  1. derives utility from a composite consumption good(+)
  2. supplies different types of labor to firms in different sectors(-)
  3. accesses to a complete set of state-contingent claims
  4. maximizes the expected utility

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \Gamma_t \left(\log(C_t) - \sum_{k=1}^{K} w_k \frac{H_{k,t}^{1+\varphi}}{1+\varphi}\right)\right]$$

     , where C_t: consumption, $H_{k,t}$: labor on sector k, β: discount factor, φ: inverse of the Frisch elasticity of labor supply, and $\{\omega_k\}_{k=1}^{K}$: relative measure of disutilities of different sectors.

  5. is under the budget constraint

$$P_t C_t + \mathbb{E}_t[Q_{t,t+1} B_{t+1}] = B_t + \sum_{k=1}^{K} W_{k,t} H_{k,t} + \sum_{k=1}^{K} \int_{\mathcal{I}_k} \Pi_{k,t}(i) di$$

     , where P_t: price level, $W_{k,t}$: wage of sector k, $\Pi_{k,t}(i)$: profits of firm i in sector k, B_t: holdings of one-period state-contingent nominal securities, and $Q_{t,t+1}$: nominal SDF.

- ☐ The aggregate consumption and price level are summarized by

$$C_t = \left(\sum_{k=1}^{K} (n_k D_{k,t})^{1/\eta} C_{k,t}^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)}$$

and

$$P_t = \left(\sum_{k=1}^{K} (n_k D_{k,t}) P_{k,t}^{1-\eta}\right)^{1/(1-\eta)}$$

, where η: elasticity of substitution b/w sectors and $D_{k,t}$: relative demand shock which is normalized to satisfy $\sum_{k=1}^{K} n_k D_{k,t} = 1$.

- □ The optimal demand for the sectoral composite good is given by

$$C_{k,t} = n_k D_{k,t} \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} C_t.$$

- □ Sectoral consumption and price level is summarized by

$$C_{k,t} = \left(\left(\frac{1}{n_k} \right)^{1/\theta} \int_{\mathcal{I}_k} C_{k,t}(i)^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)}$$

and

$$P_{k,t} = \left(\frac{1}{n_k} \int_{\mathcal{I}_k} P_{k,t}(i)^{1-\theta} di \right)^{1/(1-\theta)}$$

, where θ: elasticity of substitution within sectors.

- □ The optimal demand for the firm i's good in sector k is given by

$$C_{k,t}(i) = \frac{1}{n_k} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} C_{k,t} = D_{k,t} \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\theta} \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} C_t.$$

- □ The two remaining FOCs are

$$Q_{t,t+1} = \beta \left(\frac{\Gamma_{t+1}}{\Gamma_t} \right) \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right)$$
$$\frac{W_{k,t}}{P_t} = \omega_k H_{k,t}^{\varphi} C_t.$$

2.2 Firms

- □ Firms use sector-specific labor and intermediate goods to produce under technology

$$Y_{k,t}(i) = A_t A_{k,t} H_{k,t}(i)^{1-\delta} Z_{k,t}(i)^{\delta}$$

, wher $Y_{k,t}(i)$: production, A_t: economy-wide productivity, $A_{k,t}$: sector-specific productivity, $H_{k,t}(i)$: labor on firm i in the sector k, $Z_{k,t}(i)$: usage of intermediate goods of firm i in the sector k, and δ: elasticity of output w.r.t. intermediate inputs.

- □ The sectoral intermediate input $Z_{k,k',t}(i)$ denotes the amount of sector k' output that firm i in the sector k uses as intermediate inputs:

$$Z_{k,k',t}(i) = \left(\left(\frac{1}{n_{k'}} \right)^{1/\theta} \int_{\mathcal{I}_{k'}} Z_{k,k',t}(i,i')^{(\theta-1)/\theta} di' \right)^{\theta/(\theta-1)}$$

where $Z_{k,k',t}(i,i')$ denotes the quantity of goods that firm ik purchases from firm $i'k'$.

- □ $Z_{k,t}(i)$ is a Dixit-Stiglitz aggregator of sectoral composites with the same across-sector elasticity of substitution in consumption, η:

$$Z_{k,t}(i) = \left(\sum_{k'=1}^{K} (n_{k'} D_{k',t})^{1/\eta} Z_{k,k',t}(i)^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}$$

□ By solving the firm's cost-minimization problem, we yield

$$
\begin{aligned}
Z_{k,t}(i) &= \frac{\delta}{1-\delta}\frac{W_{k,t}}{P_t}H_{k,t}(i) \\
Z_{k,k',t}(i) &= n_{k'}D_{k',t}\left(\frac{P_{k',t}}{P_t}\right)^{-\eta} Z_{k,t}(i) \\
Z_{k,k',t}(i,i') &= \frac{1}{n_{k'}}\left(\frac{P_{k',t}(i')}{P_{k',t}}\right)^{-\theta} Z_{k,k',t}(i).
\end{aligned}
$$

□ Prices are sticky and follow Calvo model, i.e.,

$$
\begin{aligned}
P_{k,t} &= \left(\frac{1}{n_k}\int_{\mathcal{I}^*_{k,t}} P^{*(1-\theta)}_{k,t} di + \frac{1}{n_k}\int_{\mathcal{I}_k-\mathcal{I}^*_{k,t}} P_{k,t-1}(i)^{1-\theta} di\right)^{1/(1-\theta)} \\
&= [(1-\alpha_k)P^{*(1-\theta)}_{k,t} + \alpha_k P^{1-\theta}_{k,t-1}]^{1/(1-\theta)}
\end{aligned}
$$

, where $P^*_{k,t}$: common price chosen by firms that adjust at time t and $\mathcal{I}^*_{k,t}$: set of firms that can adjust prices(randomly chosen w.p. $1-\alpha_k$)

□ Firms adjust prices to maximized their expected discounted profits

$$
\mathbb{E}_t\left[\sum_{s=0}^{\infty}\alpha_k^s Q_{t,t+s}\Pi_{k,t+s}(i)\right]
$$

, where $Q_{t,t+s}$: SDF b/w time t and $t+s$

$$
Q_{t,t+s} = \beta^s\left(\frac{\Gamma_{t+s}}{\Gamma_t}\right)\left(\frac{C_t}{C_{t+s}}\right)\left(\frac{P_t}{P_{t+s}}\right)
$$

and $\Pi_{k,t+s}(i)$: expected firm ik's nominal profit at time $t+s$ conditional on the price chosen at time t

$$
\Pi_{k,t+s}(i) = P_{k,t}(i)Y_{k,t+s}(i) - W_{k,t+s}H_{k,t+s}(i) - P_{t+s}Z_{k,t+s}(i).
$$

□ The FOC is given by

$$
\mathbb{E}_t\left[\sum_{s=0}^{\infty}\alpha_k^s Q_{t,t+s}\left(\frac{P^*_{k,t}}{P_{k,t+s}}\right)^{-\theta}\left(\frac{P_{k,t+s}}{P_{t+s}}\right)^{-\eta} Y_{t+s}\left(P^*_{k,t} - \frac{\theta}{\theta-1}MC_{k,t+s}\right)\right] = 0
$$

, where

$$
MC_{k,t+s} = A^{-1}_{t+s}A^{-1}_{k,t+s}\frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta} P^{\delta}_{t+s}W^{1-\delta}_{k,t+s}
$$

is the nominal marginal cost.

2.3 Policy

□ It assume that the government neither collects taxes nor purchases goods; no influence of fiscal policy on equilibrium.

□ For monetary policy, there are two alternative assumptions:

1. nominal aggregate consumption $M_t = P_tC_t$ follows an exogenous stochastic process.

2. the gross nominal interest rate I_t is set according to the Taylor-type interest rate rule

$$\frac{I_t}{I} = \left(\frac{I_{t-1}}{I}\right)^{\rho_i} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{C_t}{C}\right)^{\frac{\phi_C}{4}}\right]^{1-\rho_i} \exp(\mu_t)$$

, where μ_t: monetary policy shock and (C, I): the zero-inflation steady state levels of consumption and interest rate.

2.4 Equilibrium

□ The market-clearing conditions are

$$\begin{aligned} B_t &= 0 \\ H_{k,t} &= \int_{\mathcal{I}_k} H_{k,t}(i)di \\ Y_{k,t}(i) &= C_{k,t}(i) + \sum_{k'=1}^{K} \int_{\mathcal{I}_k} Z_{k',k,t}(i', i)di' \end{aligned}$$

, which are for asset market, labor market, and intermediate goods market, respectively.

□ Equilibrium is solved by log-linearising the equilibrium conditions around the deterministic zero-inflation steady state.

3 Inspecting the Mechanisms

□ There are three mechanisms that allow the model to deliver differential responses of prices to aggregate and sectoral shocks:

1. pricing interactions produced by intermediate inputs
2. pricing interactions produced by labor market segmentation
3. monetary policy responses to endogenous variables

3.1 Identical Responses

□ To get the model to produce the same response of sectoral prices to aggregate and sector-specific shocks, the paper starts by abstracting from intermediate inputs($\delta = 0$), assuming perfectly elastic labor supply($\varphi = 0$) and exogenous nominal consumption m_t.

□ Under these assumptions, the model exhibits "strategic neutrality" in price setting. Firm ik's frictionless optimal price $p^{**}_{k,t}(i)$ is given by

$$p^{**}_{k,t}(i) = m_t - a_t - a_{k,t}$$

, where the RHS is nominal marginal cost.

- Since there is no effects of labor market segmentation, the wage rate, which is a key determinant of the marginal cost, are substituted out.
- Now, the RHS is simply a linear combination of exogenous stochastic processes.
- This leads firms to respond in exactly the same way to the various shocks.

3.2 Differential Responses

The Effect of Intermediate Inputs $(\delta > 0, \varphi = 0, m_t$: exogenous)

- Firm ik's frictionless optimal price $p^{**}_{k,t}(i)$ is given by

$$p^{**}_{k,t}(i) = (1-\delta)m_t - a_t - a_{k,t} + \delta p_t.$$

- The last term of RHS, δp_t, involves the aggregate price level p_t, which renders firms' pricing decisions strategic complements; this complementarity follows from the fact that the prices of intermediate goods directly affect marginal costs.
- There is a across-sector strategic complementarity; firms would like to keep their prices in line with other firms - including those in other sectors.
- Adjusting firms do not change their prices by as much in response to aggregate shocks since nominal marginal costs are 'held back' by prices that have yet to adjust.
- In conclusion, input-output linkages make slower price responses to aggregate shocks than to sector-specific shocks.

The Effect of Labor Market Segmentation $(\delta = 0, \varphi > 0, m_t$: exogenous)

- Firm ik's frictionless optimal price is given by

$$\begin{aligned} p^{**}_{k,t}(i) =& (1+\varphi)m_t + \varphi d_{k,t} - (1+\varphi)a_t - (1+\varphi)a_{k,t} - \varphi p_t \\ &+ \varphi\eta p_t - \varphi\eta p_{k,t} \end{aligned}$$

- The fifth term of RHS, $-\varphi p_t$, is reminiscent of models with an economy-wide labor market; it implies a strategic substituability in price setting.
- The last two terms of RHS, $\varphi\eta p_t - \varphi\eta p_{k,t}$, arise because sectoral labor market segmentation creates a direct dependence of a firm's marginal cost on its sectoral relative price, due to an expenditure-switching effect.
- There is two types of strategic interaction:
  1. strategic complementarity in price setting across sectors $(\varphi\eta p_t)$
  2. strategic substituability within sectors $(-\varphi\eta p_{k,t})$
- As aforementioned, across-strategic complementarity contributes to a slower response of prices to aggregate shocks. How within-sector strategic substituability leads prices to respond faster to sector-specific shocks? Consider a negative sector-specific productivity shock; lower productivity generates the need for additional labor input to compensate for the lower marginal costs in the sector; higher sectoral wage and larger, indirect impact.
- A version of the model with firm-specific labor markets fails to generate above mechanisms.

The Effect of Endogenous Monetary Policy

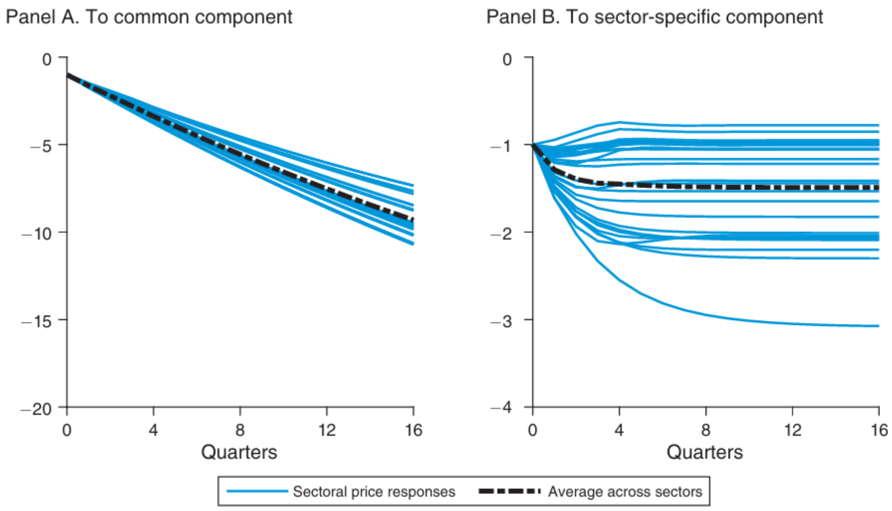
- First, nominal rigidities and staggered pricing decisions lead p_t to adjust partially in the short run.
- Second, the Taylor rule, by responding to P_t/P_{t-1}, smooths the adjustment of p_t and c_t.
- In contrast, both p_t and c_t are essentially unaffected by sector-specific shocks.

4 Quantitative Analysis

4.1 Sectoral Price Facts in the Estimated Model

□ The model is estimated with Bayesian methods, using as observables exactly the same time series used to estimate the FAVAR.

□ Thereafter, authors generate artificial data by simulating the model with parameter values fixed at the posterior mode. Based on that, FAVAR is fitted again and make below (simulated) IRF.



□ The estimated model captures the main features of the cross-sectional distribution of speeds of price responses to common and sector-specific shocks. For instance, the average speed of response to sector-specific shocks are faster and having more large cross-sectional variation.

□ On the other hand, in the IRF to sector-specific shocks, nonnegligible discrepancies b/w the model and the data are observed for a few sectors.

4.2 The Role of Each Model Ingredient

□ The all three mechanisms(in section 3.2) contribute to the slow price responses to aggregate shocks to some degree; although endogenous monetary policy plays a predominant role.

4.2.1 Intermediate Inputs and Endogenous Monetary Policy

- Without intermediate inputs/Taylor rule, the mean speed of responses to aggregate shocks is 0.364/0.736, which is significantly larger than the corresponding value 0.251 in the full model.

4.3 Labor Market Segmentation

- Without labor market segnmentation, the mean speed of responses to aggregate/sector-specific shocks is 0.270/0.745, which is larger/less than the corresponding value 0.251/0.853 in the FM.

References

[1] C. Carvalho, J. W. Lee, and W. Y. Park, "Sectoral price facts in a sticky-price model," *American Economic Journal: Macroeconomics*, vol. 13, no. 1, pp. 216–256, 2021.